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### Robustness of PD and PID Controllers used with Second-Order Processes

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#### Abstracts

Robustness is one of the requirements used in controllers and compensators design. The designs presented in the previous papers did not consider the robustness of the controller or compensator. Therefore, the objective of this paper is to investigate the robustness of PD and PID controllers when used to control second-order processes against uncertainty in the process parameters.

A variation of  $\pm 20\%$  in process parameters is considered through simulation to study its effect on the system performance parameters using the tuned PD and PID controllers. With PD controller controlling an overdamped second order process, the variation in process damping ratio has no effect on the maximum percentage overshoot, maximum percentage undershoot and settling time of the closed-loop control system. The variation of the process natural frequency produced a change of 55.5% in the settling time and zero change in maximum percentage overshoot and undershoot.

For PID controller controlling an underdamped second order process, changing the damping ratio and natural frequency of the process by  $\pm 20\%$  results in increasing the maximum percentage overshoot using the tuned controller by 18.3% and 61.3% respectively. This does not affect the maximum percentage undershoot and the settling time.

**Keywords:** Second order processes – PD and PID controllers – uncertainty in process parameters – controller robustness.

#### Introduction

Processes are subject to uncertainty in their parameters during operation. Therefore, it is worth to investigate the effectiveness of the used controllers or compensators with such uncertainty.

Hu, Chang, Yeh and Kwatny (2000) used the  $H_\infty$  approximate I/O linearization formulation and  $\mu$ -synthesis to design a nonlinear controller for an aircraft longitudinal flight control problem and address tracking, regulation and robustness issues [1]. Gong and Yao (2001) generalized a neural network adaptive robust control design to synthesize performance oriented control laws for a class of nonlinear systems in semi-strict feedback forms through the incorporation of backstepping design techniques [2]. Lee and Na (2002) designed a robust controller for a nuclear power control system. They used the Kharitonov and edge theorem in the determination of the controller which was simpler than that obtained by the  $H_\infty$  [3]. Arvanitis, Syrkos, Stellas and Sigrimis (2003) analyzed PDF controllers designed and tuned to control integrator plus dead time processes in terms of robustness. They performed the robustness analysis in terms of structured parametric

uncertainty description [4]. Lhommeau, Hardouin, Cottenceau and Laulin (2004) discussed the existence and the computation of a robust controller set for uncertain systems described by parametric models with unknown parameters assumed to vary between known bounds [5]. Dechanupaprittha, Hongesombut, Watanabe, Mitani and Ngammroo (2005) proposed the design of robust superconducting magnetic energy storage controller in a multimachine power system by using hybrid tabu search and evolutionary programming. The objective function of the optimization problem considered the disturbance attenuation performance and robust stability index [6].

Chin, Lau, Low and Seet (2006) proposed a robust PID controller based on actuated dynamics and an unactuated dynamics shown to be global ..... bounded by the Sordalen lemma giving the necessary sufficient condition to guarantee the global asymptotic stability of the URV system [7]. Vagja and Tzes (2007) designed a robust PID controller coupled into a Feedforward compensator for set point regulation of an electrostatic micromechanical actuator. They tuned the PID controller using the LMI-approach for robustness

against the switching nature of the linearized system dynamics [8]. Fiorentini and Bolender (2008) described the design of a nonlinear robust/adaptive controller for an air-breathing hypersonic vehicle model. They adapted a nonlinear sequential loop-closure approach to design a dynamic state-feedback control for stable tracking of velocity and altitude reference trajectories [9]. Labibi, Marquez and Chen (2009) presented a scheme to design decentralized robust PI controllers for uncertain LTI multi-variable systems. They obtained sufficient conditions for closed-loop stability of multi-variable systems and robust performance of the overall system [10]. Matusu, Vanekova, Porkop and Bakosova (2010) presented a possible approach to design simple PI robust controllers and demonstrate their applicability during control of a laboratory model with uncertain parameters through PLC [11].

Kada and Ghazzawi (2011) described the structures and design of a robust PID controller for higher order systems. They presented a design scheme combining deadbeat response, robust control and model reduction techniques to enhance the performance and robustness of the PID controller [12]. Surjan (2012) applied the genetic algorithm for the design of the structure specified optimal robust controllers. The parameters of the chosen controller were obtained by solving the nonlinear constrained optimization problem using IAE, ISE, ITAE and ITSE performance indices. He used constraints on the frequency domain performances with robust stability and disturbance rejection [13]. Jiao, Jin and Wang (2013) analyzed the robustness of a double PID controller for a missile system by changing the aerodynamic coefficients. They viewed the dynamic characteristics as a two-loop system and designed an adaptive PID control strategy for the pitch channel linear model of supersonic missile [14]. Pradham, Ray, Sahu and Moharana (2014) proposed a control strategy to improve the power factor and voltage regulation at disturbance supply system for more robustness [15]. Hao and Yang (2014) studied a robust adaptive fault-tolerant compensation control problem using sliding-mode output feedback for uncertain linear systems with actuator faults [16].

## Analysis

### Process

The process considered in this analysis is a second-order one having the transfer function,  $G_p(s)$ :

$$G_p(s) = \omega_{np}^2 / (s^2 + 2\zeta_p\omega_{np}s + \omega_{np}^2) \quad (1)$$

Where:  $\omega_{np}$  = process natural frequency = 5 rad/s.

$$\zeta_p = \text{process damping ratio} = 1$$

### PD controller

The PD-controller has the transfer function:

$$G_c(s) = K_{pc} + (K_d / s) \quad (2)$$

It has the 2 parameters:

- Proportional gain,  $K_{pc}$ .
- Derivative gain,  $K_d$ .

### PD controller tuning

The PD-controller was tuned to control second order processes [17]. The tuning parameters and the system performance measures are:

$$K_{pc} = 313.51$$

$$K_d = 189.36$$

$$OS_{max} = US_{max} = \text{zero}$$

$$T_s = 0.6 \text{ ms}$$

### Process uncertainty

Due to the change in the operating conditions during operation, the process is subjected to parametric changes. It is assumed that this change be as large as  $\pm 20\%$  of the assigned process parameters.

### PD controller robustness

The control system is robust when it has acceptable changes in its performance due to model changes or inaccuracy [18]. On the other hand Lee and Na add the stability requirement to the robustness definition besides the plants having uncertainty [3]. Toscano adds that the controller has to be able to stabilize the control system for all the operating conditions [19].

In this work, the robustness of the controller and hence of the whole control system is assessed as follows:

- A nominal process parameters are identified.
- The controller is tuned for those process parameters.
- A variation of the process parameters is assumed within a certain range.
- Using the same controller parameters, the step response of the system using the new process parameters is drawn and the control system performance is evaluated through the maximum percentage overshoot, maximum percentage undershoot and settling time.
- The variation in process parameters is increased and the procedure is repeated.

Application of the above procedure results in the following:

- The maximum percentage overshoot and undershoot did not change from its zero level.
- The change in the settling time increases as the change in the process natural frequency increase.
- The settling time did not change with the change in the process damping ratio.

Fig.1 shows the variation in the settling time against the variation in the process parameters

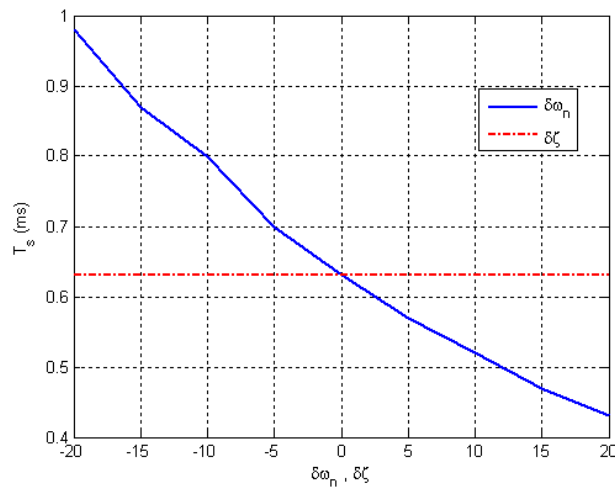


Fig.1 Effect of process parameters change of system settling time.

**PID controller controlling an underdamped second order process**

A simple tuning approach was presented resulting in one set of tuned parameters of the PID control when used with a second order process of parameters in the range [20]:

$$2.5 \leq \omega_n \leq 15 \quad \text{rad/s}$$

And  $0.05 \leq \zeta \leq 0.90$

The controller parameters were:

$$K_{pc} = 10.01023$$

$$K_i = 9.00696$$

$$K_d = 0.66375$$

To investigate the robustness of the controller, a nominal process parameter are set at:

$$\omega_n = 10 \quad \text{rad/s}$$

$$\zeta = 0.2$$

The robustness investigation procedure is applied on the resulting control system for process variation in the range  $\pm 20\%$  from the nominal values. The results are as follows:

- The maximum percentage undershoot does not change.
- The settling time does not change.
- The maximum percentage overshoot changes as both process parameters change.

The level of this change is shown in Fig.2.

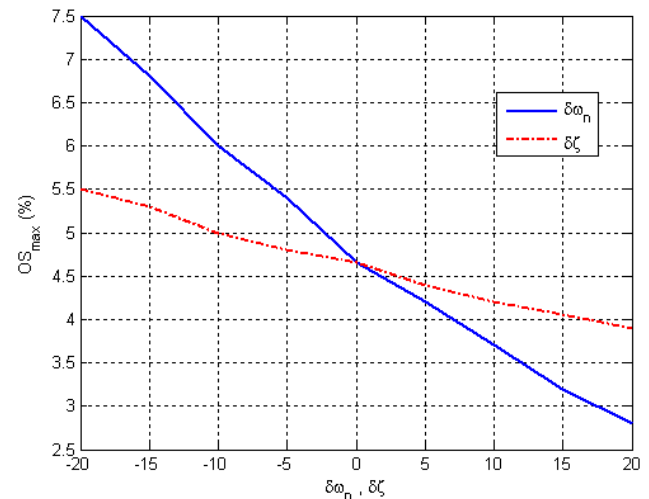


Fig.2 Effect of process parameters change of system maximum percentage overshoot.

**PID controller controlling an overdamped second order process**

A simple tuning approach was presented resulting in one set of tuned parameters of the PID control when used with a second order process of parameters in the range [21]:

$$2.5 \leq \omega_n \leq 15 \quad \text{rad/s}$$

And  $1 \leq \zeta \leq 10$

The controller parameters were:

$$K_{pc} = 29.5776$$

$$K_i = 31.4980$$

$$K_d = 31.2383$$

To investigate the robustness of the controller, a nominal process parameter are set at:

$$\omega_n = 10 \quad \text{rad/s}$$

$$\zeta = 5$$

The robustness investigation procedure is applied on the resulting control system for process variation in the range  $\pm 20\%$  from the nominal values. The results are as follows:

- The maximum percentage overshoot does not change.
- The maximum percentage undershoot does not change.
- The settling time does not change for a change in process natural frequency less than 4%.

The level of the settling time change is shown in Fig.3.

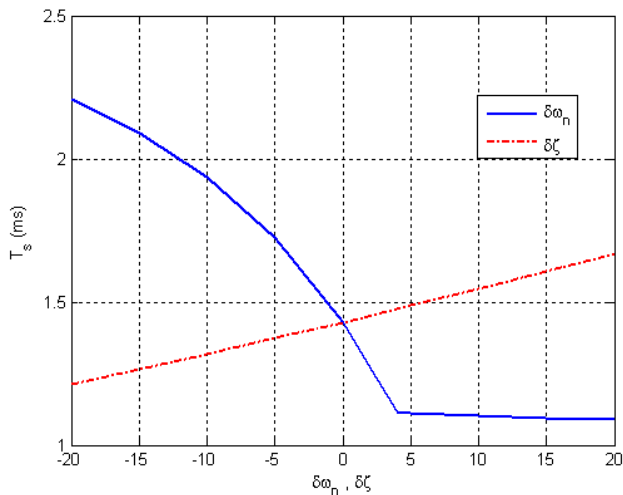


Fig.3 Effect of process parameters change on system settling time.

### Conclusions

- Variation in second-order process parameters within  $\pm 20\%$  was considered.
- Tuned PD and PID controllers are robust since they controlled the second order process for set-point change maintaining acceptable performance and stable control system for the range of parameters change.
- With PD controller, the variation in process damping ratio had no effect on the settling time of the closed-loop control system.
- With PD controller, a change of 20% in process natural frequency resulted in an increase in the settling time by 55.5% of the nominal value ( $< 1$  s settling time).
- With PD controller, a change of -20% in process natural frequency resulted in a decrease in the settling time by 31.74% of the nominal value ( $< 1$  s settling time).
- With PD controller, the change in the process parameters did not affect the maximum percentage overshoot and undershoot.
- With PID controller, the change in the process parameters did not affect the settling time and maximum percentage undershoot of the control system.
- With PID controller, a change of 20% in process natural frequency resulted in a decrease in the maximum percentage overshoot by 39.8% of the nominal value.
- With PID controller, a change of -20% in process natural frequency resulted in an increase in the maximum percentage overshoot by 61.3% of the nominal value (7.5% maximum).
- With PID controller, a change of 20% in process damping ratio resulted in a decrease in

the maximum percentage overshoot by 16.1% of the nominal value.

- With PID controller, a change of -20% in process damping ratio resulted in an increase in the maximum percentage overshoot by 18.3% of the nominal value (5.5% maximum).
- The closed-loop control system is more sensitive to the variations in the process natural frequency than the variations in its damping ratio.

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